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Uncertainty Quantification in Aeroelasticity: Recent Results and Research Challenges

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Static and dynamic aeroelasticity considerations are a particularly important component of airframe design because they often control safety and performance. Consequently, the impact of uncertainty on aeroelastic response prediction has begun to receive substantial attention in the research literature. In this paper, general sources of uncertainty that complicate airframe design and testing are briefly described. Recent applications of uncertainty quantification to various aeroelastic problems, for example, flutter flight testing, prediction of limit-cycle oscillations, and design optimization with aeroelastic constraints, are reviewed with an emphasis on new physical insights and promising paths toward improved design methods that have resulted from these studies. Several challenges and needs are explored to suggest future steps that will enable practical application of uncertainty quantification in aeroelasticity design and certification.

Introduction

ALTHOUGH computer-based analyses have resulted in increased aerospace design productivity and analytical fidelity for some time, there is still little reliance on these techniques when it comes to certification of aeroelastic stability. This is due primarily to a lack of confidence in either the analytical method or the fidelity of the model. Because avoidance of destructive aeroelastic phenomena is among the most important objectives in aircraft design, substantial flight-test resources are devoted to demonstrations of aeroelastic stability throughout the flight envelope. Freedom from both static, and dynamic, aeroelastic instabilities must be demonstrated to ensure safe operation.

Aeroelastic clearance of civil aircraft is primarily a concern during design and initial flight tests, but many military aircraft require flight tests throughout their useful life as their operational demands change, for example, through the introduction of new external stores. As a consequence, the competing forces of increasingly constrained budgets and expanding requirements for operational flexibility have generated a compelling case for revolutionizing our approach to design for and demonstration of aeroelastic stability.

This need has become increasingly pressing in recent years. Unique design concepts are being proposed to provide impressive performance gains in military applications. A common feature of these designs is that they substantially increase the potential for nonlinear behavior beyond levels that can be adequately addressed by current engineering tools and processes. These needs and concerns were the focus of a recent workshop organized by the U.S. Air Force Office of Scientific Research and the U.S. Air Force Research Laboratory.[1] The workshop addressed traditional topics, such as the basic physics and computational requirements of nonlinear aeroelasticity, but it also included sessions on model verification and validation, and the role of uncertainty quantification (UQ) in efforts to understand the physics of nonlinear aeroelasticity and to certify aeroelastic stability. The participants of the workshop developed a strong consensus that UQ must play a prominent role in the future of aeroelasticity research; notably, it was agreed that UQ could provide a common language for the promotion of communication between analysts and test personnel in a risk-informed systems engineering framework.

This paper summarizes recently published efforts to employ UQ methods in aeroelastic analysis, design, and testing. Each of the research activities described herein was undertaken with at least one of the following goals in mind: 1) improve understanding of aeroelastic performance sensitivity and variability in canonical aeroelastic systems, 2) employ aeroelastic performance criteria in a uncertainty-based design framework, and 3) accelerate aeroelastic tests and enhance the insight they produce by improvement of the use of data generated during the tests.

An important goal that does not appear to have been pursued seriously in the available literature is the selection of risk-based design criteria for aeroelastic constraints. For example, U.S. military aircraft must satisfy a 15% flutter safety margin, a requirement that seems to be essentially empirical. This requirement has been in effect since at least the early 1960s (private communication with L. Huttsell). Even though the interim has seen significant advances in computational aeroelasticity and testing. When viewed in the light of current technology, it seems reasonable to adjust aeroelasticity safety requirements, but no rational approach has been advanced for doing so. Additional comments on this important and timely topic are provided hereafter. In addition, several promising research topics are described that must be tackled to facilitate the productive use of UQ in aeroelasticity.

Role of Uncertainty in Aeroelasticity

Uncertainty in Systems Engineering

UQ is not an end in itself; rather, it is a tool that enables quantitative risk analysis (QRA), the goal of which is to provide rational guidance for design and certification decisions. In turn, risk analysis is a key step in the overarching systems engineering (SE) process.[2] Risk analysis need not be quantitative to be useful, but decision-making becomes more credible and dependable when it is. It is not the purpose of QRA to completely eliminate full-scale tests, or even traditional safety-based design constraints and philosophy; instead, QRA should be viewed as a powerful additional tool to support rational decisions.[3]

Pettit and Veley[4] discuss the analysis and allocation of risk as a primary component of the systems engineering of airframes; in particular, they focus on aeroelasticity as an important multidisciplinary phenomenon that often presents a serious SE challenge. The existence of this challenge is made manifest by the fact that inadequate aeroelastic performance often is uncovered through flight tests instead of analysis. The main purpose of the introduction of QRA into aeroelasticity is to provide designers, analysts, test personnel, program managers, and certification officials with more complete

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information both within the iterative design process and at key decision points as the program progresses.

Identification of Uncertainties

Organized identification and quantification of uncertainty sources typically involves a specific taxonomy of uncertainty. Selection of an appropriate taxonomy in a multidisciplinary field is complicated by the conflicting perspectives of the various experts from the constituent disciplines, as well as by the quality of information available in each discipline. In aeroelasticity, each of the primary disciplines, structures, aerodynamics, and controls, has a unique set of design variables, analytical methods, modeling pitfalls, and performance criteria; furthermore, the information quality and availability in the relevant disciplines often differ at a given point in the design process.

This diversity of technical priorities and vantage points has resulted in a jumble of methods and applications for the mitigation of uncertainty in aeroelastic design. This situation is common in the analysis of complex systems[5] and also reflects a continuation of the status quo in deterministic aeroelasticity. The only practical means to address these concerns in traditional airframe design has been through required margins and incrementally expanded flight tests. This paper should help to demonstrate that UQ and QRA provide a useful approach for the introduction of additional rationality and efficiency into design, testing, and certification.

In a discussion of uncertainty, we generally employ the commonly recognized classes described by Melchers[6]: aleatory or irreducible uncertainty, the most common example being randomness in a system's parameter; epistemic uncertainty, which connotes limited knowledge and is often encountered as a lack of understanding about the physics that must be modeled; and uncertainty due to human error. This classification scheme is common in structural reliability analysis. More involved taxonomies are available,[7] but the version described here is adequate for the following discussion.

Aleatory uncertainty usually is amenable to probabilistic description, which, therefore, is the primary tool in most structural reliability methods; consequently, probabilistic methods have also been the chosen approach in many of the published aeroelasticity studies that involve uncertainty. In comparison, epistemic uncertainty typically is difficult to quantify; the development of useful approaches to reflect epistemic uncertainty in complex systems is a vigorous research area, but it does not appear to have been confronted directly in the aeroelasticity literature.

This concern was sidestepped by Lindsley et al.,[8] who considered uncertainty in nominally clamped panel boundaries by adding to the model rotational springs with uncertain stiffness; thus, epistemic uncertainty about the localized mechanics of traditionally ideal boundary conditions was converted to a source of aleatory uncertainty. Of course, this is only a surrogate for model uncertainty because the insertion of rotational springs ignores the actual physical form of the panel's connection to its supports. The idea was to assume a sufficiently general approximate model that could mimic the boundary's unknown flexibility in a simple parametric form. The validity of this simplified uncertainty model can only be judged in the context of the model's goals. For Lindsley et al., this was an acceptable approximation of the actual uncertainty because they were not concerned with the the details of the stress distribution along the panel's edges.

A useful nonprobabilistic representation of uncertainty has been developed in the control systems community under the rubric of robust control[9,10] and has recently been employed to estimate aeroelastic stability.[11] The idea behind this approach, which is often referred to as μ analysis, is to partition the known parts of the system from the uncertain parts and pass information between them in a feedbacklike connection. Bounds on the possible values of the uncertain elements are either assumed or based on existing data, and the system is judged robustly stable if it can be shown to be stable for variations within the assumed uncertainty bounds. Space constraints do not permit a full description of μ analysis here, but the publications of Lind and Brenner[11] and Balas et al.[10] are good starting points for further reading. Some recent research results that involve μ analysis and aeroelasticity are summarized hereafter.

The various uncertainty classification frameworks often are not equivalent. Differences exist both in terminology and perspective. For instance, robust control theory seems not to make a clear distinction between aleatory and epistemic uncertainty; instead, all uncertainties in the plant and the input are lumped under the heading of model uncertainty. This usage differs from that employed in structural reliability, where this phrase generally denotes a subclass of epistemic uncertainty that reflects uncertainty about the physical behavior the model must represent. Commonly encountered examples include specification of failure modes (that is, what constitutes failure?), as well as any multiscale phenomenon for which several competing models can be shown partially to reproduce response quantities of interest. To some extent, what matters most is not the particular taxonomy, but the consistency and thoroughness with which it is employed. Nevertheless, the author suggests that an authoritative description of and translation between the various UQ decompositions employed by scientists, engineers, and policy makers would promote improved decision making.

Sources of Aeroelasticity Uncertainty and Their Importance

It would be impractical in the current context to develop a complete list of uncertainties in aircraft aeroelasticity. Typically, uncertainty in the physics or boundary condition models is epistemic, whereas uncertainty in the operating environment or structural properties is aleatory, except perhaps for statistical uncertainty due to limited sample sizes. The relative importance in airframe design of these and other sources of uncertainty depends on several considerations, such as the 1) vehicle's layout, materials, load paths, and environment; 2) vehicle's class and purpose; 3) actual usage of each aircraft, for example, some pilots stress their aircraft more than others; 4) risk aversion of the users; 5) amount of design experience with component-level technologies and their integration into complex systems; and 6) level of experience with newer manufacturing processes. These considerations are relevant to the entire field of airframe design and to aeroelasticity in particular.

As unique design concepts and structural technologies are proposed with increasing frequency, it is becoming clear that the interaction of system nonlinearities with the various sources of uncertainty must be understood more thoroughly. In turn, this requires reliable and accurate approaches for the prediction of nonlinear aeroelastic phenomena. Consider limit-cycle oscillations (LCO), induced by Hopf bifurcations, that have been observed in several aeroelastic systems (see Refs. 1 and 12). The response associated with a given Hopf bifurcation strongly depends on whether the bifurcation is supercritical or subcritical, as shown in Fig. 1. The introduction of uncertainty into this problem results in an uncertain bifurcation point and, therefore, a level of risk that depends critically on the nature of the bifurcation, as well as the amount and type of uncertainty.

Hopf bifurcations can occur in aeroelastic models that are nonlinear just in the structural stiffness operator (for example, panel LCO), but, from the perspective of computational cost, model and mesh uncertainty, and physical complexity, nonlinear aerodynamics is, in most cases, the critical component of computational aeroelasticity that must be enhanced to enable dependable predictions of aeroelastic response and variability. Some related research topics are suggested hereafter.

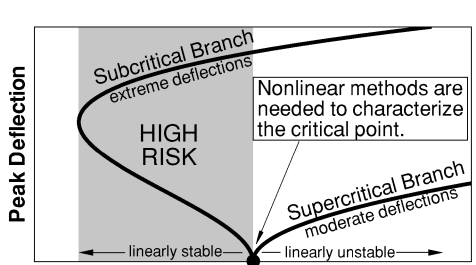


Fig. 1 Subcritical and supercritical Hopf bifurcations.

UQ can also assume great prominence in aeroelastic tailoring. Two primary contributing factors are 1) the relatively high variability in the properties of composite materials and 2) the fact that model and parametric uncertainty can lead to unexpected and significant aeroelastic performance degradation in practice.[13] The latter is simply a specific realization of the common observation that optimum designs tend to be fragile in the sense that they perform poorly at off-design conditions.[14]

Anyone conducting an analysis, design, or testing process that makes explicit use of UQ should consider the points described in this subsection. Current program management and certification frameworks, such as those employed by the U.S. Air Force (USAF), depend on iterative application of systems engineering methods to manage risk and promote safety while attempts are made to meet performance goals. Multiple reviews conducted during system definition, design, and testing help to ensure that risks are identified and managed, but airframe design and certification as a whole includes little to no rigorous UQ and, therefore, little QRA. Pettit and Veley[4] discuss the rationale for the inclusion of QRA in airframe design. They also consider specific technical and framework issues that must be addressed to enable QRA to serve as a primary decision-support tool in the overarching systems engineering process. Much of their discussion considers the special aspects of aeroelastic risk in this context.

Probabilistic Aeroelasticity Analyses

Flutter of Aircraft Lifting Surfaces

Flutter is a dynamic aeroelastic instability that can destroy a lifting surface's structure if the resulting response is not sufficiently restricted, perhaps by nonlinear damping or stiffness in the system. Classical flutter prediction with linear structural and aerodynamics models can be pursued through various algorithms,[15,16] the underlying goal being to solve a parametric or pseudo-eigenvalue problem that yields the flight speed and oscillation frequency at which flutter first occurs. Specialized iterative algorithms are required because, except in the simplest case of quasi-steady aerodynamics, the aerodynamic forces depend nonlinearly on the oscillation frequency. Details of the performance of flutter analyses are available in the aeroelasticity references cited earlier.

Few published efforts focus on classical flutter in a probabilistic context. The basic problem was described by Poirion,[17] who employed a first-order perturbation method to solve for the probability of flutter given uncertainty in the structural mass and stiffness operators, which were represented by a linear finite element model. Poirion observed that, although this perturbation formulation of the stochastic eigenvalue problem produced an explicit estimate of the flutter probability, application to a realistic system model generated results that compared poorly with Monte Carlo simulation. The quality of Poirion's results were also difficult to evaluate because no details of the unsteady aerodynamics model were provided.

A later paper by Poirion[18] took a more general approach in the formulation of linear aeroservoelasticity in a probabilistic context. The equations of motion were projected onto the structural modes to reduce their dimension, linear aerodynamics was assumed to apply, and a standard frequency-domain formulation[16] was employed. Finally, the aerodynamics operators were represented through rational function approximations. Poiron also summarized particular methods for the simulation of Gaussian and non-Gaussian stochastic processes. His paper can be viewed as complementary to the discussion herein because it focused primarily on linear systems, random loads dues to gust, and uncertainty in the control inputs. It did not explicitly discuss uncertainty in the system operators or physics-based modeling of stochasticity in the flow.

LCO of Airfoils

Pettit and Beran[19] studied a nonlinear incarnation of the standard aeroelastic system consisting of an airfoil with pitch α and plunge h degrees of freedom (DOF), as shown in Fig. 2. Unsteady aerodynamic forces were represented by the R. T. Jones approximation (see Ref. 20) of the circulatory lift. Their deterministic structural and aerodynamic model was an extension of that employed by Lee et al.,[21] in which the plunge DOF had linear stiffness, but the pitch DOF included a third-order stiffness term in addition to the linear component. The third-order term k_{α_3} modeled aeroelastic LCO induced by a supercritical Hopf bifurcation. Pettit and Beran included a fifth-order pitch stiffness term k_{α_5}, so that properly chosen stiffness coefficients would induce a subcritical Hopf bifurcation, which is more dangerous than the supercritical bifurcation because of the potential for an abrupt increase in LCO amplitude associated with subcritical bifurcations. This concept is shown qualitatively by the bifurcation diagram in Fig. 1.

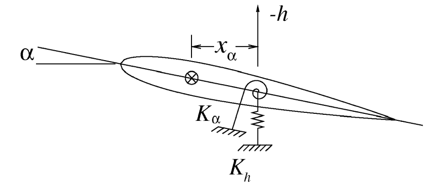


Fig. 2 Two-degree-of-freedom airfoil.

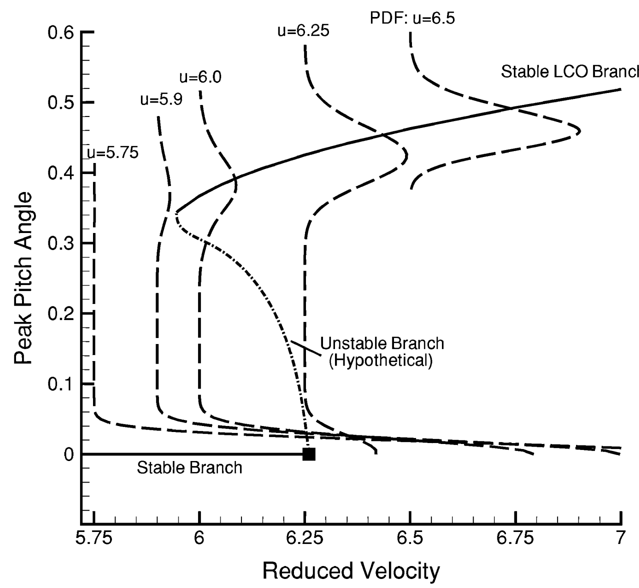


Fig. 3 Response of pitch-and-plunge airfoil: ——, LCO amplitude of baseline airfoil and - - - -, estimated PDF from MCS at each dynamic pressure.

Standard Monte Carlo simulation (MCS) and time integration were employed to quantify LCO amplitude variability induced by assumed parametric variability in the initial pitch angle α_0, k_{α_3}, and k_{α_5}. These variables were assumed to be independent. The resulting random bifurcation diagram is shown in Fig. 3, which shows the estimated LCO amplitude probability density function (PDF) for several reduced velocities. Because the aeroelastic model was rather elementary and no attempt was made to base the structural coefficients or their assumed variability on a physical system, only qualitative conclusions can be drawn from Fig. 3. It is intended mainly to provide an informative presentation of the MCS results for a nonlinear aeroelastic system.

Foremost among the benefits of the representation in Fig. 3 is the ability to estimate the probability that LCO will exceed a specified threshold at a given reduced velocity. Information presented in this manner could, in the future, be the basis for a risk-based flutter design criterion. For example, deterministic analysis of the baseline system predicts zero steady-state response, that is, a stable focus for $u = 5.75$, which is approximately 8% below the linear instability at $u = 6.26$ and is also below the lower limit point on the unstable LCO branch ($u = 5.95$); however, MCS predicted a small but nonzero probability of encountering LCO at this reduced velocity. The risk induced by the parametric uncertainties is reflected clearly in the bimodal shape of the amplitude PDF at these reduced velocities.

Pettit and Beran[19] also showed a scatter plot of the random system parameters that illustrated the separation of the realizations into classes based on whether or not LCO was observed. In doing so, the plot clearly displayed the regions of the input parameter space that should be avoided to prevent LCO. It was apparent that the occurrence of LCO in this system depended strongly on α_0 and that this dependence was essentially symmetric with respect to zero pitch. The third-order stiffness term had a secondary effect on the occurrence of LCO, and the fifth-order term primarily governed the amplitude of any LCO that does develop.

Note that a standard deterministic analysis would correspond only to one point on the scatter plot. Even if a particular combination of system parameters and initial conditions does not exhibit LCO, the relative risk of the design cannot be definitively assessed without the context provided by the clusters of points in the scatter plot.

An interesting issue not addressed by Pettit and Beran is the basis for the assumption of independence between k_{α_3} and k_{α_5}. This was done purely for convenience, but the author can find no a priori physical justification for doing so. Two questions immediately present themselves. First, how does an assumed dependence between k_{α_3} and k_{α_5} propagate through the assumed physics model? That is, would it make much difference if their dependence were modeled incorrectly? Second, do the physics of nonlinear stiffness imply a particular type of correlation?

The first question can be answered simply by running additional simulations, but the latter is of a more fundamental nature. Conceptually, this problem can be restated as follows. Suppose that a spring design were developed to follow a certain nonlinear stiffness characteristic, that multiple samples of this spring were produced for quality assurance tests, and that the stiffness characteristic of each individual spring was modeled as the sum of a set of nonorthogonal monomials,

$$k(x) = \sum_{i=0}^{n} k_i x^i$$

Under these conditions, the coefficients $\{k_i\}$ for each sample spring can be viewed as realizations of random variables $\{K_i\}$, whose interdependence is unknown a priori. The question then becomes, is there a physical basis for the determination of whether or not these variables are dependent and, if so, what is the qualitative nature of their dependence? This seems to be a fundamental question, but the author has been unable to find a relevant discussion in the literature. On this point, the author can only offer the general observation that if two random variables are dependent, they usually are functions of one or more random variables that do not appear in the model.

LCO of Panels

Several conference and journal papers have addressed the sensitivity of panel flutter and panel LCO to uncertain system parameters. This problem actually involves what is, in practice, a relatively benign nonlinear behavior in that it does not induce catastrophic failure; nevertheless, it has been a productive vehicle for exploration of the use of UQ in aeroelasticity.

Liaw and Yang[22,23] examined flutter and LCO of laminated plates and shells with uncertainties in several structural and geometric parameters. Their work appears to represent the first published application in an archival journal of a stochastic finite element formulation for this problem. Their approach involved a second-moment, perturbation-based stochastic finite element model. They presented several figures that detailed the effects of variability on the likely range of responses, but because they employed a second-moment formulation, quantification of output variability was limited to the corresponding second moment also.

Depending only on second moments can be a serious limitation in the study of any nonlinear system because the output variables generally are non-Gaussian; a specific example was provided earlier in Fig. 3. The practical implication of this important fact is that higher moments or, ideally, complete density functions are needed to quantify output variability for nonlinear systems. This does not mean that second moments are always insufficient; for example, the amplitude PDFs shown in Fig. 3 are reasonably Gaussian in appearance for reduced velocities well above the bifurcation point. It is fair to conclude that a quasi-Gaussian representation would be adequate for these conditions, but it must be remembered that this conclusion can only be justified by comparison with the more complete MCS results discussed earlier.

Lindsley et al.[24] also studied LCO of panels with spatial variability in the modulus of elasticity, $E(x, y)$, but they employed MCS to better quantify the range of response variability in this nonlinear system. They limited their analysis to square panels and isotropic materials, but also included the influence of nonideal boundary conditions (BCs) to measure the relative importance of uncertainty in material properties and BCs. The elastic modulus field was assumed to follow the assumption of a spectral density function based on an exponentially decaying spatial correlation function. Monte Carlo realizations were generated under the assumption that the phase of each spectral component was an independent uniform random variable and by addition of the resulting components. Newland[25] provides a lucid description of this common approach to simulating isotropic random fields. Figure 4a shows the concept of an ensemble of panels generated through MCS of $E(x, y)$. The upper contour plot shows a single realization of this random field, and the lower plot demonstrates that an ensemble of 200 panel realizations yields an average panel for which $E(x, y) \approx E_0$, the specified average value of the random field.

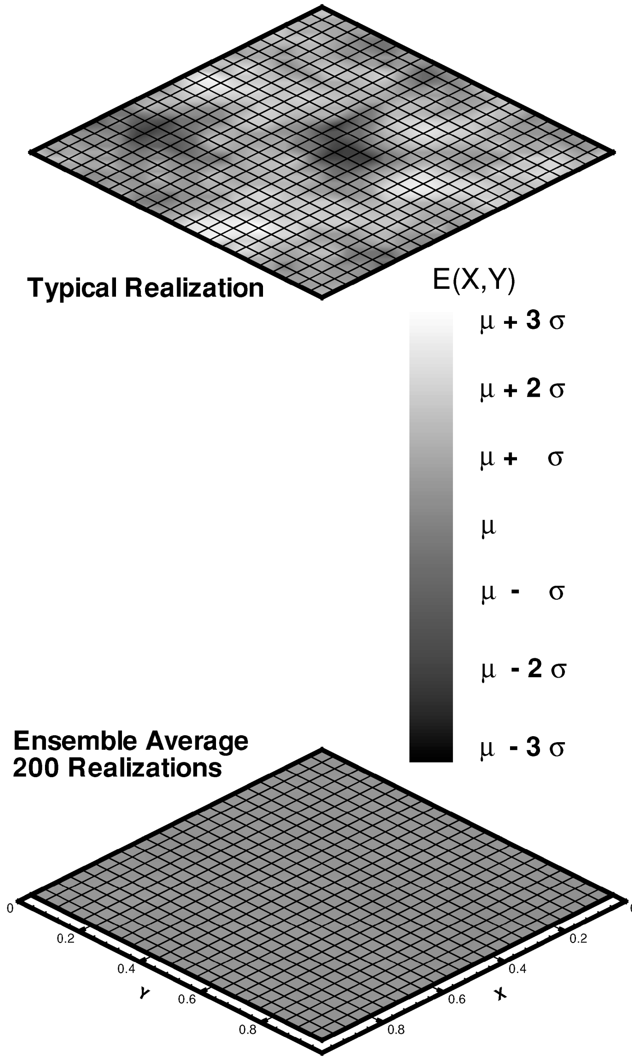
Figure 4b shows the output variability in the bifurcation diagram for LCO amplitude vs dynamic pressure, with a separate nonparametric PDF of response amplitude based on 200 panel realizations at each of several dynamic pressures. At dynamic pressures well above the baseline bifurcation point ($\lambda \approx 858$), Young's modulus variability had a relatively minor effect in that it simply added a random component to the steady-state LCO amplitude, but at $\lambda = 860$, modulus variability controlled the panel's long-term response, with the realizations approximately evenly split between LCO and zero deflection. Perhaps most important is the result that several realizations had a response amplitude much larger than the deterministic case.

Figure 5 shows the proper orthogonal decomposition[26–28] of the elastic modulus fields that resulted in either an LCO or in no LCO. These plots strikingly demonstrate that LCO typically was associated with relatively low stiffness near the three-quarter-chord region (measured from the panel's leading edge). Also, Fig. 6 shows the parametric influence of nonideal BCs, where distributed torsion springs along each boundary were used to model flexibility in ideally clamped boundaries. At $\lambda = 850$, which is clearly below the deterministic bifurcation point, a moderate amount of boundary flexibility, parameterized by β, was seen to induce LCO.

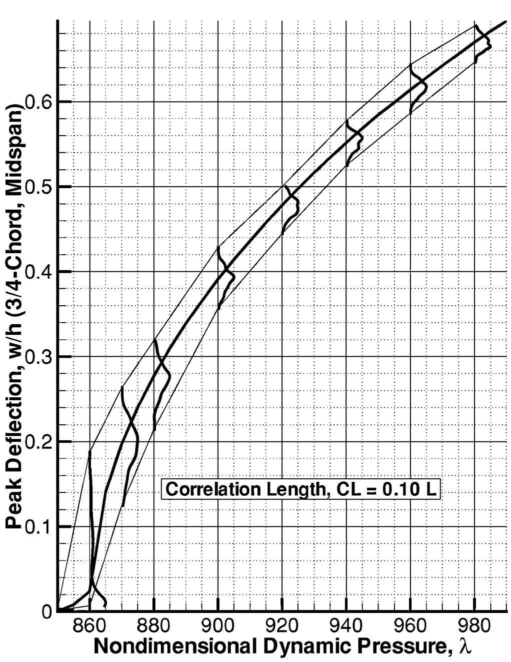
More recent work by Lindsley et al.[8,29] has extended the study of LCO of uncertain panels. In Ref. 8, they showed that the occurrence and amplitude range of LCO at a given dynamic pressure are relatively insensitive to the assumed ratio l_c of correlation length of the elastic modulus random field to panel side length, at least for $0.10 < l_c < 0.30$. They also examined the influence of the assumption of combined independent variations in $E(x, y)$ and the rotational springs along each boundary.

Lindsley et al.[29] also studied aerothermoelastic effects through the addition of an uncertain thermal expansivity field, $\alpha(x, y)$, which was modeled by alteration of the baseline biaxial prestress induced by an assumed temperature change. Qualitative response classes are summarized in Fig. 7, which shows the baseline response, that is, $E(x, y) = E_0$ and $\alpha(x, y) = \alpha_0$, as a function of compressive prestress and dynamic pressure. The variety of responses exhibited by the baseline system are typified by the plots in Fig. 8, which shows the response for several dynamics pressures and a dimensionless prestress of $-8\pi^2$.

Realizations of $E(x, y)$ and $\alpha(x, y)$ were computed to measure their influence on the dynamic behavior of the panel. A wide range of behavior was observed, with several realizations falling into each of the categories depicted in Fig. 7. In particular, stochastic results were generated for prestress and dynamic pressure values of $-8\pi^2$ and 230, which result in a very weak attractor at zero deflection (Fig. 8). The vast majority of the realizations resulted in zero steady-state deflection, but a small number of realizations produced limit-cycle or chaotic behavior; three of these are depicted in Fig. 9.



a)



b)

Fig. 4 MCS of stochastic panel: a) ensemble of panel Young's modulus realizations and b) bifurcation diagram, - - - -, LCO amplitude of baseline panel, and ——, estimated PDF from MCS at each dynamic pressure, along with their associated envelope.

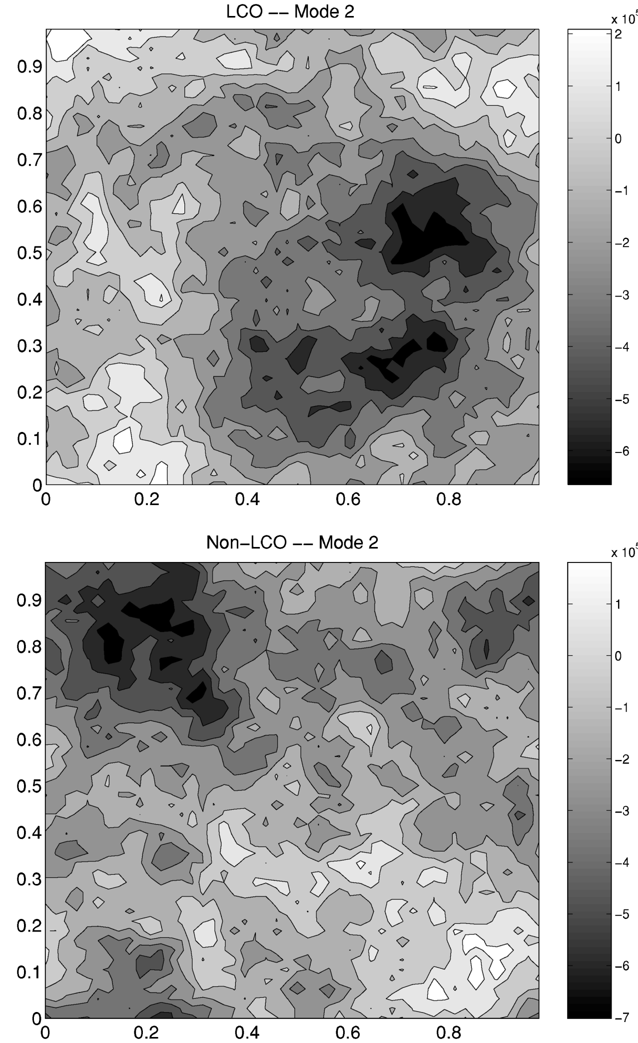


Fig. 5 Modes of proper orthogonal decomposition of Young's modulus field for LCO and non-LCO cases.

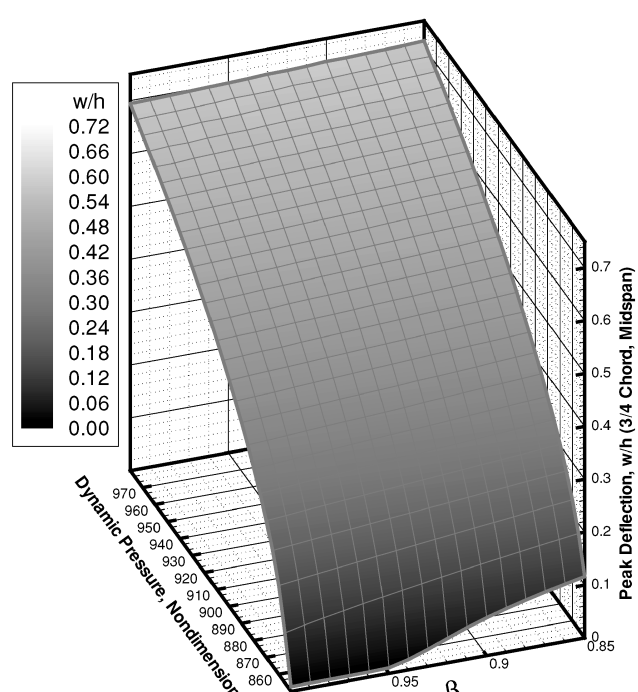


Fig. 6 Parametric influence of flexible boundaries on panel LCO amplitude.

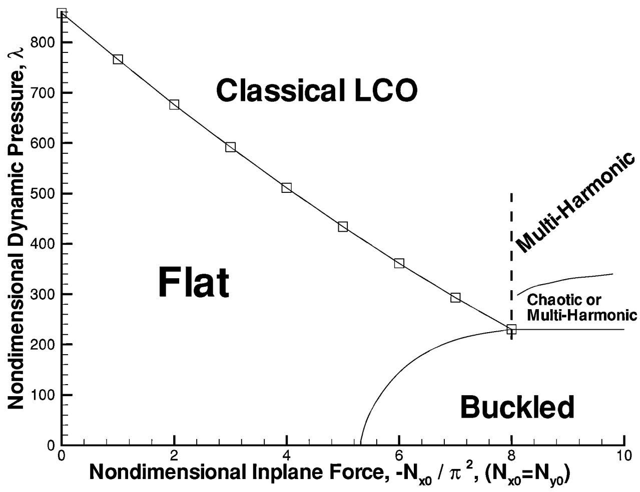


Fig. 7 Baseline aerothermoelastic stability diagram.

A comparison of Figs. 8 and 9 shows that the inclusion of spatially variable thermoelastic properties in the panel can induce dynamic bifurcations of similar severity to those caused by significant changes in the dynamic pressure of the baseline system.

Probabilistic Design for Aeroelasticity

Reliability-Based Optimization

Space restrictions preclude a detailed description of standard structural reliability methods, but a few key points are described here to assist the uninitiated reader. Melchers[6] provides a complete description of the theory. For simplicity, we assume that only a single mode of failure, for example, flutter, LCO, or aileron reversal, is under investigation. In practice, the reliability of the complete aeroelastic system, which is subject to many possible failure modes, can only be quantified through a system reliability analysis.[6] This is necessary to account for correlation between the various failure modes. The theory summarized here can be viewed as a component of the entire system reliability problem.

The fundamental premise of structural reliability analysis is that the space of uncertain design variables and system parameters x is divided into two regions by a limit-state function or failure surface, $g(x) = 0$. The limit-state function is a level curve of the response or performance function, such that $g \leq 0$ corresponds to states of failure. The probability of failure is given symbolically by

$$P_f = \int_{\Omega_f} p(x)\,\mathrm{d}x \tag{1}$$

where $p(x)$ is the probability density of the performance function and $\Omega_f = \{x : g(x) \leq 0\}$ denotes the failure region.

The most commonly employed method for approximation of structural reliability is the first-order reliability method (FORM). FORM is really a generic name for a family of methods that use the reliability index β as a surrogate for P_f. Under very restrictive assumptions,[6] $P_f = \Phi(-\beta)$, where $\Phi(\cdot)$ is the cumulative distribution function of a zero-mean Gaussian random variable with unit variance. More generally, this equality is only an approximation, the quality of which is sensitive to nonlinearity of the failure surface and to the degree by which the random variables depart from Gaussian behavior.

Reliability-based optimization (RBO) is generally formulated as a typical optimization problem in which the objective function is a familiar choice, for example, structural weight, and constraints are imposed on the reliability index for each failure mode. This description is overly simplistic; in particular, it omits correlation between failure modes, which, as noted earlier, are the subject of system reliability. This simplicity is imposed for the purpose of economy in the current presentation and should not compromise the reader's understanding of this section.

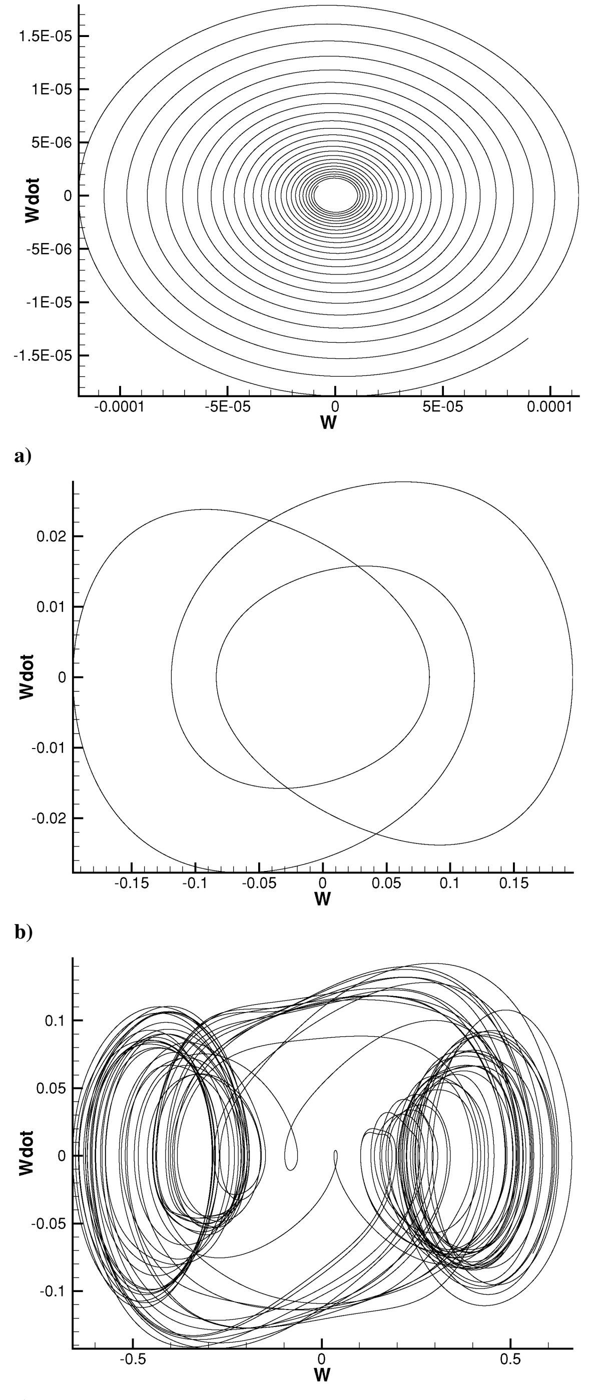


Fig. 8 Phase-plane plots of baseline panel for several dynamic pressures and dimensionless prestress of $-8\pi^2$: a) $\lambda = 230$, b) $\lambda = 235$, c) $\lambda = 250$, and d) $\lambda = 290$.

RBO of preliminary wing designs for aeroelastic performance has been pursued recently, with published efforts beginning to appear in the mid-1980s. One of the earlier studies is by Yang and Nikolaidis,[30] who employed FORM in conjunction with system reliability optimization to design for gust loads a simplified wing structure composed of a segmented box beam. They compared their probabilistic design with a deterministic design based on Federal Aviation Administration (FAA) regulations and showed that optimization based on system reliability could be used to obtain a lighter wing structure with the same implicit reliability as the design based on FAA requirements or a higher reliability wing with the same weight as the FAA wing; however, their formulation could not be employed directly in practical design owing to the elementary nature of their structural model.

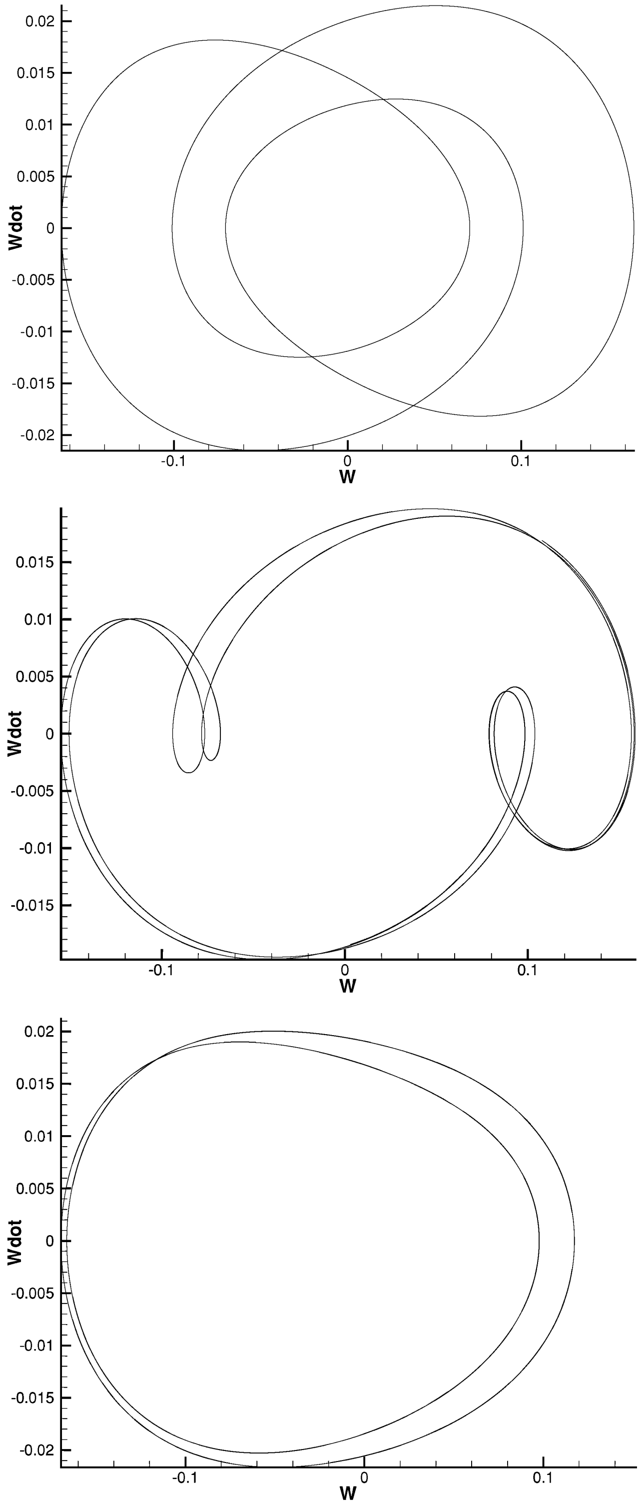


Fig. 9 Representative phase-plane plots of aerothermoelastic response of the stochastic panel for prestress and dynamic pressure values of $-8\pi^2$ and 230, respectively.

A general concern that has received little attention in aircraft RBO studies is relevant here also: Yang and Nikolaidis did not address any empiricism or modeling limitations implicit in the assumptions on which the FAA-specified design factors and requirements were based. Given that the existing FAA criteria for continuous gust design were proposed in the early 1970s (Ref. 31) and that the supporting data, assumptions, and modeling methods are even older, it is reasonable to suggest that modeling advances and improvements in data acquisition and processing technology during the interim justify the revision of safety requirements to ensure that they reflect current knowledge and analysis capabilities. This would impact both

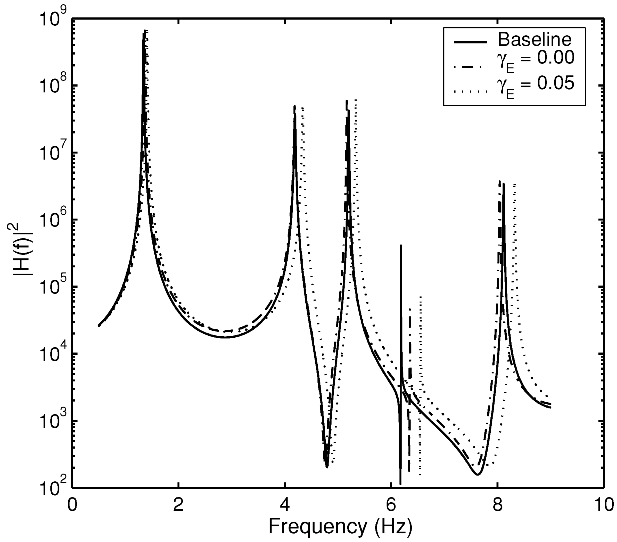


Fig. 10 Squared magnitude of FRF for root bending moment: curve marked $\gamma_E = 0.00$ is for zero variability in Young's modulus and curve marked $\gamma_E = 0.05$ is for a Young's modulus coefficient of variation of 0.05.

deterministic and reliability-based design methods, especially because the latter are often developed as probabilistic translations of the limit states prescribed in deterministic safety requirements.

Pettit and Grandhi[32,33] developed an RBO framework by combining ASTROS, a deterministic, linear finite element based structural optimization system that includes linear aeroelastic analysis, with DOT® to control optimization, and several MATLAB® functions that were written to implement a FORM that uses a two-point adaptive nonlinear approximation[34] of the limit-state function to accelerate convergence of the reliability index estimate. They performed RBO of a relatively simple, fighter-like wing structure with uncertain element thickness values and elastic modulus to achieve specified reliability indices in several performance functions, including static deflection, aileron effectiveness, and gust response. Mean thickness values of the structural elements were taken as design variables, so that the mean weight served as the objective function. Redistribution of structural mass by the optimizer produced designs with improved aeroelastic performance reliability and relatively small weight penalties.

Figure 10 shows the baseline and optimized frequency response functions (FRF) of the root bending moment. The first optimum design compensated for variability in the element thickness values, whereas the second optimum design also accounted for Young's modulus randomness. The observed changes in the FRF with the additional uncertainty agree with intuition because they demonstrate that additional uncertainty in the structure's stiffness must be accommodated by an increase in the mean stiffness, that is, a rise in the mean modal frequencies of the structure.

A primary impact of this study is that it helped to demonstrate the readiness of RBO methods to be applied in preliminary design to ensure reliable aeroelastic performance. Moreover, comparison of the reliability-based design results with deterministic designs based on traditional criteria could provide insight into how existing safety factors account implicitly for parametric variability. A study of this nature could perhaps spawn efforts to improve the rational basis of safety factors and required stability margins, but this path for implementing uncertainty-based design criteria has certain shortcomings, for example, see Ref. 35 and the references cited therein, that could restrict its utility.

Allen and Maute[36] recently extended RBO for aeroelastic performance to include computational fluid dynamics (CFD) in place of the doublet-lattice method for the computation of unsteady pressures. They employed a finite volume based Euler flow solver coupled with a linear finite element structural solver to model the physics of the problem; in addition, they employed an adjoint formulation to compute sensitivities, which helped to constrain the relatively high computational cost of this step of the FORM algorithm. Their introduction of high-fidelity aerodynamics modeling adds an additional level of realism to the uncertainty-based aeroelastic design problem and further supports the conclusion that RBO is ready to begin being transitioned to industrial aeroelastic design applications.

Allen and Maute[36] also commented on the potential importance of accounting for parameter uncertainty in aeroelastic tailoring to achieve optimum cruise performance, which depends on an accurate representation of the design's static aeroelastic performance. They cited earlier results by Kuttenkeuler and Ringertz,[13] who suggested that the solution of an aeroelastic tailoring problem is rather sensitive to uncertainty.

As noted, successful studies of this nature show that there is no fundamental technical issue that prevents the use of RBO for aeroelastic design based on traditional analysis tools. The primary impediments can be found in the inertia of current practice and the lack of an accepted framework for including formal risk quantification throughout the airframe certification process. The author has recently cowritten a paper[5] that begins to address these concerns directly, but substantial work is still needed to realize the promise of risk quantification in aeroelastic design; in particular, this will require the cooperation of program managers and other decision-makers responsible for judging the progress of a given design through the many formalized stages of the systems engineering process.

Aeroelasticity Applications of Robust Control Concepts

Nonprobabilistic Methods for Analysis and Design

Given the maturity of nonprobabilistic robust control theory, for example, μ analysis,[9,37] and the importance of flight-control systems in ensuring aeroelastic stability, it is natural that this theory has yielded perhaps the most exercised UQ methods in aeroelasticity. A comprehensive review of this work would require substantial space, and so only a few relevant references are cited to help the reader gain a feeling for the basic aspects of this approach, as well as its pros and cons relative to the probabilistic methods described earlier.

Karpel et al.[38] developed an aeroservoelastic design process that includes robust control theory to accommodate structured uncertainty in the plant. They applied their process to design a flutter suppression system for a fighter wing structure that included four control surfaces and a tip missile with uncertain inertial properties. The open-loop aeroelastic system was stable for the entire range of uncertainties in the missile's inertial properties, but the controller in the closed-loop system, that is, including the actuated control surfaces, could not be designed through standard methods to meet robust performance specifications. They showed that an additional structural design cycle could be performed to yield a new closed-loop system with the desired level of robustness. Note that this application of UQ for the structure is fundamentally different from those described earlier because it captured uncertainty at a relatively high level, that is, in the state-space representation of the system, instead of the physical level, which includes the material properties, dimensions, and boundary conditions.

Chavez and Schmidt[39] observed that employment of robust control theory to analyze multidisciplinary systems requires an adequate uncertainty or variation model for each of the component analysis disciplines. This can be a compromising factor in practical applications because each discipline is subject to a variety of uncertainties that often are not characterized in a consistent manner or with uniform information quality. They addressed this concern through employing a systems approach to characterize the anticipated uncertainty in finite dimensional, linear, time-invariant models of flexible aircraft arising from unsteady aerodynamics, truncation of structural modes, and uncertain mass and stiffness properties. Chavez and Schmidt also considered the possibility of interaction between the rigid-body dynamics and structural modes of highly flexible aircraft, which is an issue of potentially high importance in certain airframe design concepts.

Lind and Brenner have published a series of papers[40–42] and a monograph[11] that develop the application of nonprobabilistic robust control methods to flight testing of aeroelastic stability. Although there is no fundamental impediment to the use of this approach in a purely analytical mode, in which it has been shown to compute nominal flutter margins that closely match those predicted by traditional flutter prediction methods,[11] the primary applications have been in the use of wind-tunnel, ground, and flight test data to update and validate aeroelastic models. Consequently, their work is reviewed separately in the following subsection.

Uncertainty in Aeroelastic Tests

Comprehensive wind-tunnel and flight-test programs are required to demonstrate that aeroelastic instabilities do not develop within the operational envelope. Of particular concern is the sometimes abrupt occurrence of flutter when the damping in a particular aeroelastic mode decreases suddenly. Because the flight conditions that induce flutter are difficult to predict precisely, flight-test programs generally are conducted to ensure that the flutter point is approached gradually, a practice that increases their duration and cost. These difficulties have motivated recent work to improve the integration between flight-test data and model predictions to increase the overall efficiency and dependability of flutter margin estimation. The purpose of the published efforts has not been to promote fundamental change in aeroelastic certification philosophy, which continues to be based on a traditional safety margin; instead, nonprobabilistic UQ methods have been employed to improve the estimation of the true margin for the particular aircraft and flight conditions being tested.

Lind and Brenner[11] illustrate in detail the implementation and utility of the μ method; consequently, the present discussion is primarily an attempt to place this approach within the context of other uncertainty-based aeroelastic methods. Among the most attractive features of the μ method is the rigorous mathematical framework in which the concept of closeness between model and reality is enforced. In this approach, the uncertainty and modeling errors in system operators, inputs, and outputs are represented by a set of norm-bounded operators $\mathbf{\Delta}$. Flutter speeds or dynamic pressures are computed that are robust in the sense that, given the available input data, the system will be stable for modeling errors within $\mathbf{\Delta}$. As a result, the definition of an appropriate $\mathbf{\Delta}$ is crucial to the success of this approach: A $\mathbf{\Delta}$ that is too large will result in an overly conservative margin that could restrict the permitted flight envelope unnecessarily, whereas underestimating $\mathbf{\Delta}$ will lead to an overestimate of the relative aeroelastic stability.

Lind and Brenner have made significant progress by applying μ analysis to compute robust flutter speeds that represent worst-case flight conditions with respect to assumed forms of uncertainty in the plant and excitation. A key feature of their μ analysis framework is its ability to accommodate both model and flight-test information in an integrated manner. This feature is shown schematically by the flowchart in Fig. 11, which summarizes a reasonably general version of the algorithm. This process includes the option of filtering the

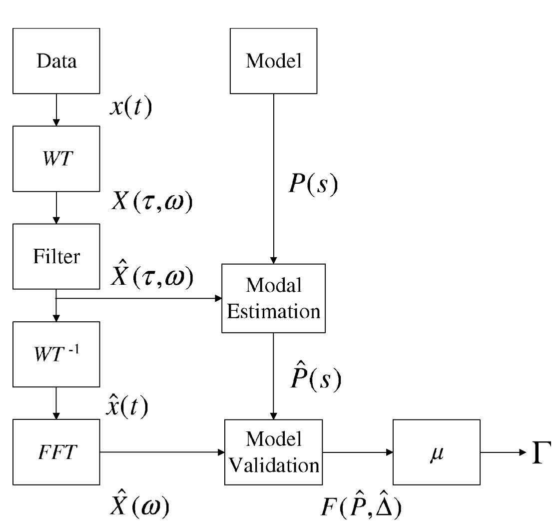


Fig. 11 Flowchart of μ-analysis framework for estimation of robust flutter boundaries (R. Lind, private communication, Jan. 2003).

experimental data $x(t)$ with a wavelet transform (WT) to reduce noise and unmodeled dynamics in the data and also to estimate modal parameters.[43] The nominal linear aeroelastic model, $P(s)$, is updated to $\hat{P}(s)$ based on the measured data, which are then used to determine how close the estimated model is to reproducing the measured dynamics. The output Γ is the estimated robust stability margin.

Although the μ method provides a comprehensive and useful framework for aeroelastic stability estimation, it does have restrictions that could be important in certain applications. First, because it is inherently nonprobabilistic, it cannot take full advantage of probabilistic information that might be available. Second, the μ framework is based entirely on linear operators. If the system in question cannot be represented adequately by linearized operators, the μ-method can accommodate the nonlinearity only in a coarse manner by associating sufficient uncertainty with the linear system model to cover errors that result from unmodeled nonlinear dynamics. The practical implications of these limitations depend on the aircraft and flight conditions in question, but, in general, they imply that μ analysis provides only a qualitative measure of aeroelastic risk. In this respect, a robust flutter margin is like a traditional flutter margin in that as the margin is approached, the likelihood of an encounter with an instability cannot be assessed quantitatively.

A notable shortcoming in the early implementation of the μ method for estimation of flutter margins is that it was not based on match-point flutter solutions. Recently, Lind[42] described a refinement of this method that does compute robust match-point flutter margins on a model that includes a theoretical mass matrix, a stiffness matrix derived from ground vibration tests, and aerodynamic forces computed with a doublet-lattice program. He also suggested that the method could be extended readily to form a refined version of the flutterometer,[41] which provides an online approach for robust estimation and update of flutter speeds during flight tests.

An important recent step is the improvement of estimates of model uncertainty through the use of Volterra kernels to isolate the linear component of the flight data (see Refs. 44 and 45). This approach has been particularly successful in reducing the potentially high conservatism of robust flutter margins. This is illustrated in Fig. 12, which compares recent flutterometer results based on Volterra kernels with flutter speed estimates computed with an earlier realization of the flutterometer, as well as a traditional, damping–extrapolation method.

Borglund[46] also recently employed μ analysis in his study of the robust aeroelastic stability of a flexible slender wing model in a low-speed wind tunnel. The model included a controllable trailing-edge flap that could be used in conjunction with an existing optical measurement system to complete a closed-loop feedback control system for aeroelastic control studies. Borglund noted that the utility of a robust stability analysis for aeroservoelastic design depends on the availability of realistic uncertainty descriptions for the various components of this multidisciplinary problem; this observation agrees with the preceding comments regarding the choice of $\boldsymbol{\Delta}$. He focused on uncertainty in the unsteady aerodynamic forces, which generally are the most difficult part of the problem to model accurately. The μ validation test recommended by Lind and Brenner[11] was used to estimate the amount of uncertainty in the aerodynamic model, which was represented by uncertain coefficients in Fourier expansions of the spanwise loading for each aerodynamic mode shape. The size of the aerodynamic uncertainty model was restricted by the admission of uncertainty only in the lowest Fourier coefficients, which corresponded to aerodynamic mode shapes driven by the lowest structural modes. This was deemed an acceptable approximation because these modes dominated the structural motion.

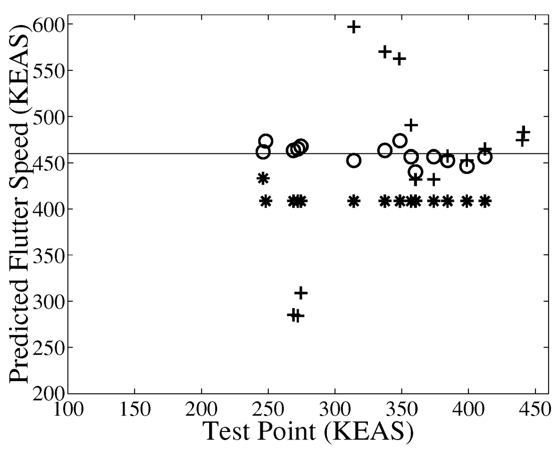


Fig. 12 Flutter speeds predicted by damping extrapolation, +; original flutterometer, ∗; and updated flutterometer, ○ (R. Lind, private communication, Jan. 2003).

Research Needs

The ultimate goal of what can be called stochastic computational aeroelasticity (SCAE) is the efficient and accurate computation of uncertainty-based performance estimates like those shown in Fig. 3, but for realistic structures and flight conditions. Several issues remain to be tackled before this goal can be approached with a reasonable hope of success. Many of these issues are technical and will require fundamental advances in the computational modeling of uncertain nonlinear aeroelastic systems. However, an equally important impediment is the current lack of an well-defined role for quantitative risk assessment in aeroelasticity certification.

This section emphasizes technical challenges that ought to be tackled in future research, but it also introduces a few philosophical and framework points. It is hoped that the ideas presented here will help to motivate future efforts to restructure aeroelasticity certification as a more formally risk-informed decision-making process. Pettit and Veley[4] provide a more extensive discussion of this complex and multi-faceted problem.

Aeroelastic System and Model Uncertainties

General Observations

The importance of system and model uncertainty in the prediction and understanding of aeroelastic response variability is amplified in many applications by the degree of nonlinearity exhibited by the system within the operational envelope. It was noted earlier that aerodynamic model uncertainty often is substantially greater than uncertainty in the structural model or parameters; furthermore, aerodynamics often is the dominant source of nonlinear behavior in aeroelastic systems. Nevertheless, the majority of published probabilistic applications continue to focus on uncertainty in structural properties because these are easier to model and because structural solvers generally are orders of magnitude faster than fluid solvers. Given that quantification of purely structural uncertainty is much more highly developed and that there is an abundance of literature available on this broad topic, the research needs cited here focus on aerodynamics as well as the aeroelastic consequences of structural uncertainty.

Approaches that do explicitly recognize aerodynamics uncertainty have been based primarily on robust control methods. This approach is useful within the restrictions noted earlier and discussed in greater detail by Lind and Brenner, but there is some concern that the robust control methods are not adequate for the systems that are the most likely to experience destructive nonlinear aeroelastic phenomena, for example, subcritical Hopf bifurcations. This suggests that a more comprehensive physics-based approach to modeling aeroelastic uncertainty should be developed for use in more extreme applications.

Key sources of uncertainty in the aerodynamics models employed in computational aeroelasticity remain to be explored more thoroughly than in current practice. Perhaps the most commonly recognized are the standard sources of model uncertainty encountered in deterministic simulations, such as grid convergence, approximation of boundary conditions, and turbulence models. This observation should be considered in the light of the recent work by Walters and Huyse,[47] which is briefly described hereafter. Uncertainty in the initial conditions of both the structure and the incoming flow also deserves substantial consideration because their selection, which is often based on the analyst's judgment, can govern the qualitative accuracy of computational aeroelasticity predictions. This point becomes especially important in aeroelastic systems that exhibit subcritical Hopf bifurcations because their stability near the bifurcation point is sensitive to the level of disturbance.

Variability in the freestream encompasses the traditional domain of gust analysis.[48] This area has, for at least 50 years, involved some probabilistic content in the form of traditional spectral analysis and random vibrations of linear systems, but the methods used to accommodate gust loads in airframe design still produce what are effectively deterministic design loads. The standard approach combines simple estimates of the gust load spectral density with empirical safety factors. This patchwork method predates even the wide availability of computational linear aeroelasticity tools for preliminary design, much less more sophisticated and accurate flow solvers.

Weishaupl and Laschka[49] demonstrated the additional insight available just from the employment of deterministic computational aerodynamics when unsteady loads caused by a nonuniform freestream are modeled. They developed an approach for including a simple shear layer or ring vortex in the freestream to study the influence of the nonuniform flow features. However, this important first step did not include any statistical basis for the choice of parameters of these coherent flow events, nor did it account for aeroelastic interaction. It is recommended that this work be extended to include aeroelastic interaction and the spatio-temporal statistical properties of atmospheric variability. The relevance of work in this area is underlined by the increase in popularity of uncrewed aerial vehicles (UAVs) for surveillance, which often have flexible, high-aspect ratio wings that are sensitive to coherent structures in the atmosphere.

Series Expansion of Stochastic Processes in Aeroelasticity

An exciting new area of research combines stochastic spectral projection with traditional CFD models. Published approaches generally employ the polynomial chaos expansion (PCE), the basics of which are described in the context of solid mechanics by Ghanem and Spanos.[50] PCE now is also being used to model uncertainty in fluid properties, initial conditions, and boundary conditions. The PCE decomposes a random process in terms of deterministic functions of space and time and orthogonal functions of random variables, that is,

$$u(x, t, \theta) = \sum_{i=0}^{n} \hat{u}_i(x, t)\psi_i(\theta) \tag{2}$$

where θ denotes the random dimension of the problem and n controls the truncation of the series. The orthogonality of the $\{\psi_i\}_{i=0}^n$ is with respect to the probability measure of the underlying random variables. When this expansion is inserted into the governing equations and a Galerkin projection is performed on the $\{\psi_i\}_{i=0}^n$, the $\{\hat{u}_i\}$ become the solution variables.

A recent PCE application that could eventually bear on aeroelasticity involves the stochastic eigenvalue problem. Ghosh et al.[51] employed PCE to quantify variability in the natural frequencies and modes of a system of nominally identical coupled oscillators, but with uncertain spring stiffness values. The baseline system included periodic boundary conditions, which resulted in repeated modes. A key product of this formulation is a set of random variables whose statistics measure the typical similarity of the stochastic modes to the baseline modes and thereby provide a possible foundation for modal truncation of stochastic structural dynamics models. Given the fundamental importance of eigenvalue problems in aeroelasticity, the extension of this formulation to flutter prediction ought to be pursued.

Walters[52] and Walters and Huyse[47] have recently pioneered work in stochastic CFD that could have direct application to SCAE. They studied the use of PCE to predict variability in a model system governed by the general Burger equation with uncertain viscosity. An important result is that the treatment of the boundary conditions and the quality of the grid affected the error convergence as a function of the order of the PCE; that is, a direct connection was observed between the order of stochastic approximation of the field variables, the approximation of the boundary conditions, and the spatial discretization. This seems to contradict the common assumption that an acceptable deterministic model is automatically suitable for stochastic simulation.

Walters[52] also solved the two-dimensional Laplace equation with geometric uncertainty in the boundary of the computational domain. This study showed that it was necessary to represent the grid transformation metrics with polynomial chaos expansions. Because of the importance of high-quality moving grids in computational aeroelasticity, this conclusion suggests an important area for future research.

Other groundbreaking recent efforts that employ PCE for CFD include those of Le Maitre et al.,[53] Xiu and Karniadakis,[54] and Xiu et al.[55] These methods are still in their infancy but show great promise for integration of UQ with CFD and, eventually, SCAE. Consequently, the author and a colleague have recently initiated research in this direction.

Multidisciplinary Considerations

An important source of uncertainty in multidisciplinary models is the coupling of discipline- or process-specific models. Although this could be viewed as an algorithmic issue, which would properly place it in the following subsection, it seems appropriate to include it here because of the potential for confusion of the spurious effects of poor coupling with theory-driven physics. This is already a widely recognized concern in deterministic aeroelastic models,[56–59] and its importance can only be exacerbated by UQ efforts that depend on multiple runs of coupled models. In addition, there are UQ-specific issues that have recently been observed and should be considered when complex, multidisciplinary systems, whose physics can be simulated only through the use of high-performance computing, are modeled. Romero et al.[60] recently discussed these concerns in the context of the weapon systems exposed to fires, but the author is not aware of any similar SCAE studies. This area should be investigated with the goal of clear documentation of uncertainty modeling pitfalls relevant to high-performance computation of nonlinear aeroelastic phenomena.

Computational Approaches and Efficiency

It is commonly recognized that UQ demands substantial computing power that often increases the requirements of a single deterministic solution by an order of magnitude or more. Therefore, future progress in UQ for complex systems will likely be facilitated by a combination of progressive increases in available computational power with efficient modeling techniques both in the deterministic and stochastic subspaces of the analyses.

Reduced-order models of the aerodynamics[27,28,61,62] are currently under consideration for SCAE to improve the efficiency of time-accurate models. This work is at a relatively early stage of development, but shows some promise for practical applications in the future. Several research teams are actively engaged in further developing these methods. The reader is directed to Refs. 27, 28, 61, and 62 for additional details, as well as a comprehensive set of relevant publications. For linear systems governed by stochastic partial differential equations, a stochastic subspace projection method has recently been developed and shown to have attractive theoretical and performance advantages over solution algorithms that employ perturbation or Neumann expansions. Nair and Keane[63] reviewed the underlying theory and recent applications in structural dynamics.

Another potential source of improved computational efficiency in stochastic modeling is the diminishing marginal payoff associated with increasingly fine discretization of the structural and fluid domains. Marczyk[64] noted that this practice is yielding rapidly diminishing returns in some applications. This conclusion is based on the observation that meshes often are refined to a level of numerical precision not justified by the uncertainties in the system parameters, initial conditions, or boundary conditions; in other words, milking additional precision from a deterministic model is not justified when the numerical tolerances drop below the level of natural variability in the system. This wasted computational power, which typically is used for extremely refined deterministic point solutions, could more profitably be invested to perform multiple realizations with less-refined meshes. The author is not aware of any general methods that have been developed to identify a suitable break-even discretization threshold, but Marczyk[64] provided some elementary specific examples to illustrate the point.

It remains to be clearly established which UQ applications will benefit most from the use of nonlinear physics-based methods in conjunction with reduced-order models. This depends in part on which type of output information is required of the analysis, but this class of methods could be too computationally expensive for many linear aeroelasticity applications. It is more likely that they are appropriate for nonlinear aeroelasticity applications in which bifurcations are a dominant concern. Consider as a case in point the bimodal response density functions exhibited by these systems near the bifurcation point, for example, Fig. 3. This bimodality can be interpreted as a lack of robustness in the system under these conditions; that is, small changes in the bifurcation parameter or initial conditions can produce disproportionately large changes in the response. For large models, polynomial response surfaces often are used as surrogate system models in conjunction with FORM or some other approximate reliability method to decrease the computational expense.[65] However, simple response surfaces clearly cannot represent bifurcations and, therefore, will be inadequate in an approximate reliability analysis of this class of system. This suggests a profitable role for reduced-order models like those cited earlier, which better respect the essential physics of the system.

Monte Carlo simulation could be used as it was in the airfoil and panel examples described earlier, but in conjunction with a reduced-order representation of the physics that still captures the essential nonlinearities. Stochastic projection methods also could provide a reasonable compromise between probabilistic fidelity and computational expense, but this is an unexplored area in aeroelasticity. The author and a colleague have recently initiated a research effort in this direction. Less expensive sampling approaches, such as importance sampling,[6,67] could be used to avoid some of the computational expense of standard MCS, but there is no evidence in the literature that they have been used in stochastic aeroelasticity. Melchers[6] observes that this approach also has a particular advantage over standard MCS if the sampled random variables are not independent, but it must be remembered that the efficiency of importance sampling depends on specification of a sampling distribution that concentrates samples in regions of the random variable space that include the highest probability portions of the failure surface. The difficulty of doing so depends on the nonlinearity of the failure surface; consequently, the choice of an appropriate sampling distribution for bifurcating aeroelastic systems is an open question because the typical nature of the failure surface has not been adequately explored.

An alternative approach could include employment of a more democratic sampling approach, such as Latin hypercube sampling,[66,67] to explore the topography of the response. An importance sampling distribution then could be chosen with some confidence that it will cover an appropriate sampling region, or the analyst might decide that only standard MCS will provide adequate coverage.

Role of Stochastic Analysis in Decision Making

Airframe reliability in most failure modes is currently enforced through empirically derived safety factors and required margins. The associated design and certification procedures tend to be cumbersome to implement and account rather indiscriminately for uncertainties and their concomitant risks. These procedures can be made to suffice for conventional design concepts and structural technologies; however, their dependence on dated methods can be a serious impediment to the introduction of innovative designs, materials, and assembly technologies for which little history is available to justify the selection of safety factors and the prediction of likely failure modes.[35]

Furthermore, as noted earlier in the discussion of reliability-based optimization, some unknown portion of existing safety requirements exists to compensate for the well-known limitations of traditional analytical methods. Although higher-fidelity analysis tools continue to permeate design environments, there is no accepted process in place for incorporation of the resulting improved prediction quality in safety requirements. Note that this would not necessarily lead to a reduction of safety factors. If the existing empirically based safety factor implicitly incorporates the assumption that a given traditional analysis tends to be conservative, but the newer and more accurate analytical tool removes this conservatism, the consequence would be that the employment of a traditional safety factor in concert with the new analysis approach will actually result in a lower level of safety.

Future designs based on unique structural concepts (e.g., the joined wing[68,69]) or the incorporation of potentially disruptive technologies (e.g., adaptive structures and distributed health monitoring systems) will likely require improved methods for the allocation of system-level aeroelastic risks (e.g., flutter, divergence, and control-surface reversal) to the structural, controls, and aerodynamic components of the system. Even current systems could benefit. For example, what portion of the probability of LCO is associated with uncertainty in the mass distribution of a wing-tip store? Even if adequate computational models were commonly available for store-induced LCO, critical questions like this could not be answered dependably through traditional design frameworks, which include only standard parametric sensitivity analysis.

Each analysis discipline involves multiple uncertain factors; some of these are shared with the other disciplines, whereas others are unique to the discipline in question. The fidelity of the discipline-specific analysis methods and the quality of available discipline-specific information generally are time dependent and not equivalent at any single time in the design and testing process. Moreover, the transition to QRA as an aid to decision-making could demand substantial changes throughout the current airframe systems engineering process.[4] Given the complexity of this problem, failure to develop appropriate decision-making methods and criteria could render the associated risk-informed design problems infeasible or impractical.

Extensive research is needed to address this critical question in anticipation of the transition of cutting-edge structural technologies from research to technology demonstration and production. A potentially useful starting point could be to study how the current flutter margin requirement is allocated implicitly between parametric uncertainty and unmodeled structural and aerodynamic nonlinearities. This is a reasonable goal given existing analysis methods and could be a first step toward the rational selection of risk-based aeroelastic design criteria.

This discussion regarding justifiable precision in the presence of uncertainty also suggests a more general need: modeling guidelines and best practices based on UQ demands in addition to traditional criteria. Effective implementation of uncertainty-based design criteria and decision processes for aeroelasticity or any other area of airframe design will require coordinated use of uncertainty modeling criteria to ensure that limited analysis and test resources are invested appropriately to maximize understanding of the system's performance. The objective should be to promote early recognition, characterization, and prioritization of uncertainty sources. With these thoughts in mind, it must be recognized that the act of increasing reliance on analysis to support design and certification decisions institutes greater demands for high-quality analyses; consequently, the transition to a decision framework that is strongly dependent on stochastic analysis will require highly trained analysts who understand the potential pitfalls of stochastic simulation, more rigorous software verification and validation, and frameworks that are intimately tied to established best practices.

Conclusions

This paper has summarized key points in the quest to understand how various forms of uncertainty influence the prediction and understanding of aeroelasticity, which is one of many components of airframe design and certification that could benefit greatly from widespread use of uncertainty analysis. The author's recent conference paper[35] provides a broad review of the current status and some potential benefits of the employment of UQ throughout the airframe design process.

Recent research in elementary nonlinear aeroelastic systems, such as the work cited herein, has shown that relatively minor levels of variability in system parameters, loads, and boundary conditions can induce significant changes in the system stability. Aeroelastic sensitivity to these sources of variability is likely to increase as the aircraft design community strives to achieve historically significant improvements in the performance, range, endurance, and operational flexibility of future designs. Revolutionary advancements in these areas will require innovative aerodynamics, structural, and control systems, as well as new materials tailored to the demands placed on the structure.

The active aeroelastic wing[70] is an early example of an aeroservoelastic system that breaks with traditional aeroelastic design methods and philosophy; anticipated operational requirements call for design concepts that will demand even greater changes in design processes and perspectives. These advancements will not be possible within existing design and decision frameworks, which are historically based and, therefore, inherently biased against the efficient introduction of new technologies in all of the disciplines that influence aeroelastic performance. Because UQ provides a consistent basis for assessment of system performance and technological risks, it should be a cornerstone of aeroelasticians' efforts to stimulate fundamental advances in airframe design.

Finally, a concerted effort is needed to transition existing UQ methods from the research community to industry. The interested reader is referred to Ref. 4 for a detailed aeroelasticity-centric discussion of key systems engineering issues that must be confronted to enable this transition. In addition to the points described therein, the author suggests that an authoritative description of, and translation between, the various UQ decompositions employed by scientists, engineers, and policy makers would promote improved communication and decision making. It is recommended that the many technical societies involved with aerospace technology should jointly promote an effort to unify UQ terminology in the future. To some extent, this unification is already being pursued in the context of specific applications (e.g., systems-level simulations and validation of computational mechanics models), but it ought to be established at a higher level to form a well-defined context for future research and process development.

Acknowledgments

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